MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

EMT2056 MATHEMATICAL TECHNIQUES

(ME)

30 MAY 2016
02:30 P.M – 04:30 P.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **3** pages including this cover page with **4** questions only.

2. Attempt all four questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Print all answers in the answer booklet provided. Do show all relevant steps.

4. Programmable calculator is not allowed.

Question 1

(a) Find the value(s) of β such that the following system of linear equations has an infinite number of solutions.

$$\begin{aligned} x+\ y+\ z &= 1 \\ x+2y+\ 4z &= \beta \\ x+4y+10z &= \beta^2 \end{aligned}$$

[8 marks]

(b) Determine the general solutions for the following system of linear first-order differential equations:

$$\begin{aligned} y_1' &= 2y_1 + 3y_2 \\ y_2' &= y_1 + 4y_2 \end{aligned}$$

[17 marks]

Question 2

(a) Consider the following Runge Kutta method of order three for approximating the solutions of the differential equation $y' = f(x, y)$.

$$\begin{aligned} y_{i+1} &= y_i + \frac{1}{6}(k_1 + 4k_2 + k_3) \\ k_1 &= h f(x_i, y_i) \\ k_2 &= h f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right) \\ k_3 &= h f(x_i + h, y_i - k_1 + 2k_2) \end{aligned}$$

Use the above formula with step size $h = 0.2$ to approximate $y(0.4)$ when

$$y' = y - x^2, \quad y(0) = 2.$$

[13 marks]

(b) Find the first two iterations of the Gauss-Seidel method with $\mathbf{x}^{(0)} = \mathbf{0}$ in approximating the solutions for the following system of linear equations.

$$\begin{aligned} 3x_1 - \ x_2 + \ x_3 &= 6 \\ 2x_1 + 6x_2 + \ x_3 &= 6 \\ -\ x_2 + 2x_3 &= 6 \end{aligned}$$

[12 marks]

Continued ...

Question 3

(a) Evaluate the integral $\iint_R (x+y)(2x+y)\,dA$ using change of variables, where R is the region bounded by the lines $x+y=-2$, $2x+y=0$, $x+y=0$, $2x+y=2$.

[13 marks]

(b) Given the integral

$$\int_0^1 \int_0^1 \int_0^{1-y} x\,dx dy dz$$

(i) Sketch the region of integration in xyz-space. [4 marks]

(ii) Change the order of integration from $dxdydz$ to $dydxdz$. Then, evaluate the integral. [8 marks]

Question 4

(a) Evaluate the surface integral, $\iint_\sigma x^2+y^2\,dS$, where σ is the surface of the cone $z^2=2(x^2+y^2)$ bounded by $z=0$ and $z=2$.

[10 marks]

(b) Find directional derivative of $f(x,y)=x^2+2y^2$ at the point $P(1,2)$ in the direction $\underset{\sim}{u}=\frac{2}{3}\underset{\sim}{i}+\frac{1}{3}\underset{\sim}{j}$.

[5 marks]

(c) Evaluate the integral $\oint_C \left(2z^2\,\underset{\sim}{i}+x\,\underset{\sim}{j}-y^2\,\underset{\sim}{k}\right)\cdot d\underset{\sim}{r}$, where C is the circle $x^2+y^2=1$ in the xy-plane with counterclockwise orientation looking down the positive z-axis.

[10 marks]

End of Paper